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Nonlinear observer-based control with application to an anaerobic digestion process

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A B S T R A C T

This paper deals with the design of a new observer based control strategy for an Anaerobic Digestion (AD) process to track a desired reference trajectory. The target is to control the biogas production, and to subsequently integrate the biogas plants in a virtual power plant. The used model is a two-steps (acidogenesis-methanogenesis) mass balance nonlinear, which is a widely used AD process model. Since the AD processes experience a lack of physical sensors, an exponential nonlinear observer is designed to estimate and update the internal state of the process. Based on the estimated states, a state feedback controller is used to track any given and admissible desired trajectory with respect to an $\mathcal{H}_\infty$-optimality criterion. The observer-based controller parameters are designed by solving two (implicitly)-independent LMI conditions obtained by rigorous mathematical arguments based on a judicious use of Young relation and a reformulation of the Lipschitz property. A simulation study is provided to illustrate the validity and effectiveness of the proposed theoretical results.

1. Introduction and preliminaries

1.1. Introduction

Anaerobic Digestion (AD) is a biological process through which the organic waste is converted into a mixture of gaseous that is called biogas. The latter can be valorized as electricity, heat, biofuel, or it can be injected into the natural gas grid. Considering electricity and natural gas grids, the potential of biogas is substantial, and allows for the integration of higher shares of variable renewable energy sources in the electricity grid. Hybrid systems are currently being internationally developed. They combine biogas plants with weather dependent technologies for balancing the grids [18]. However, due to the complexity of the AD process and fluctuations of input influents [12], continuous monitoring and design of sophisticated control strategies are required for the safety of the process, and improving efficiency.

With the aim to enhance the AD process performances, different control strategies have been proposed in the literature. Often, the adopted control strategy depends on the model complexity, the available measurements and the design criteria (pollutant minimization, product maximisation or digester stabilisation). With respect to the proposed control for reduced analytical AD models (two-steps “acidogenesis-methanogenesis” models), we may cite the control of bicarbonates alkalinity concentration in the digester through the addition of stimulating substrates to the process [11,14,16]. In [11], the linearizing control was employed to enhance the biogas quality, while in [14,16], the input to output linearizing control was used to stabilize the digester. In the first and second control strategies, the magnitude of the added input was assumed very small so the general structure of the model does not change. In other words, the added input affects only the state for which it has been added (for example, added alkalinity affects only dynamics of the alkalinity concentration). Such an assumption makes the control simpler to design, especially the input to output linearizing control, which requires a nonlinear transformation of the system. Indeed, the mentioned assumption relaxes complexity of the required transformation. However, neglecting the effect of an additional input in the model dynamics is not very convenient. In parallel, we may also cite the Model Predictive Control (MPC) that has been proposed in [15] to control the biogas production for a demand-driven electricity production. The idea is to optimize the plant feeding according to a fluctuating timetable of energy demand. The control was applied to a full-scale research plant, and

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has shown satisfactory results. However, the analytical stability proof of the closed loop system is yet difficult to prove.

Combining the ideas from [11,14,16] and [15], where the alkalinity addition is employed to stabilize the reactor and enhance the biogas quality, and the plant feeding is used to optimize the production, we will propose a control strategy to track an admissible reference trajectory. This trajectory mimics a desired biogas production that corresponds to the operator objectives. Moreover, the control strategy takes into account all the process dynamics including those that are not accessible for measurement. Indeed, it is well known that the key parameters of the AD process, such as bacteria, are difficult and costly to measure; and this is the reason why state observers have been designed repeatedly in the literature. The asymptotic observer [2] is quite simple to design, and does not require knowledge of some specific nonlinear functions (kinetic functions). Nevertheless, it is very sensitive to the model uncertainties, and its convergence rate depends on the operational conditions. Thus, it has been extended to interval observers [4], which have the advantage of using reliable measurements that are nonlinear functions of the state vector. The interval observers estimate the intervals where the system state is laying when the model is subject to large uncertainties. However, generally the rate of convergence is partially tunable, and it is not easy to exploit the intervals for control [6]. Furthermore, the Kalman filter [10,17] has been proposed to enhance the convergence rate of estimation error towards zero. Although the Kalman filter has shown suitable results in different chemical applications, unfortunately, the convergence of estimation errors to zero is not guaranteed. Within this context, we may cite the high gain observer [8,9], which has a fast convergence rate to the model state variables; but its synthesis is complex, and it is very sensitive to noise [13]. In this paper, we will use the LMI-based nonlinear state observer proposed in [7] due to its robustness, systematic implementation and fast convergence. Then, the estimated states are used in a feedback control, which accounts for all the process dynamics. The synthesis of the observer-based controller parameters is done by solving two different LMI conditions. From feasibility point of view, this kind of separation principle for nonlinear Lipschitz systems (proposed in this paper) provides enhanced LMIs, when compared to the existing LMI techniques in the literature. These LMIs ensure the exponential convergence of the estimation error towards zero, and guarantee that the tracking error satisfies a given $\mathcal{H}_\infty$ criterion. The proof of convergence is done by using rigorous mathematical arguments that are easy to follow. To get enhanced LMI conditions, we used the famous Young's relation in a convenient way, and the standard Lipschitz condition is exploited in a different form to take into account all the components of the nonlinearity of the system.

The remainder of the paper is organized as follows. In Section 2, we pose the problem of observer-based reference tracking control. Then, in Section 3, we provide new LMI conditions which ensure the exponential convergence to zero of the estimation error and the $\mathcal{H}_\infty$ asymptotic stability of the tracking error. Furthermore, in Section 4, we give a detailed description on how to apply the designed control to the AD process model. We also run a numerical simulation to emphasize the effectiveness of the proposed control scheme. Finally, we conclude the paper in Section 5.

1.2. Notation and preliminaries

This section is devoted to the notation used throughout this paper and some useful lemmas.

1.2.1. Notation

The following notation will be used throughout this paper:

- $(\star)$ is used for the blocks induced by symmetry;
- A^T represents the transposed matrix of A;
- $\mathbb{I}_r$ represents the identity matrix of dimension r;
- for a square matrix $S, S>0\,(S<0)$ means that this matrix is positive definite (negative definite);
- the set $Co(x, y)=\{\lambda x+(1-\lambda)y, 0\leq\lambda\leq 1\}$ is the convex hull of $\{x, y\}$;
- $e_s(i)=\big(\underbrace{0,\ldots,0,\overbrace{1}^{i\text{ th}},0,\ldots,0}_{s\text{ components}}\big)^T\in\mathbb{R}^s, s\geq 1$ is a vector of the canonical basis of $\mathbb{R}^s$.

1.2.2. Useful lemmas

We present hereafter two lemmas, which will be used to get tractable and less conservative LMI conditions.

Lemma 1.1 ([19]). *Let φ: $\mathbb{R}^n\to\mathbb{R}^q$ be a differentiable function. Then, the following items are equivalent:*

- *φ is a globally γ_φ-Lipschitz function;*
- *For all $x, y\in\mathbb{R}^n$, there exist $z_i\in Co(x, y)$, $z_i\neq x$, $z_i\neq y$, scalar constants a_{ij}, b_{ij}, and functions ψ_{ij}: $\mathbb{R}^n\to\mathbb{R}$ such that:*

$$\varphi(x)-\varphi(y)=\sum_{i,j=1}^{q,n}\psi_{ij}(z_i)\mathcal{H}_{ij}\Big(x-y\Big) \tag{1}$$

and

$$a_{ij}\leq\psi_{ij}\Big(z_i\Big)\leq b_{ij}, \tag{2}$$

where

$$\psi_{ij}(z_i)\triangleq\frac{\partial\varphi_i}{\partial x_j}(z_i),\ \ \mathcal{H}_{ij}=e_q(i)e_n^T(j).$$

Notice that we have introduced this lemma in order to simplify the presentation of the design methodology. Indeed, throughout this paper, we will exploit (1) and (2) instead of a direct use of the Lipschitz property.

Lemma 1.2 ([19]). *Let X and Y be two matrices of appropriate dimensions. Then, for any given symmetric positive definite matrix S of appropriate dimension, the following inequality holds:*

$$X^TY+Y^TX\leq\frac{1}{2}\Big[X+SY\Big]^TS^{-1}\Big[X+SY\Big]. \tag{3}$$

This lemma will be very useful to enhance the feasibility of the LMI conditions.

2. Formulation of the observer-based tracking problem

One class of control design problems consists of planning and following a reference trajectory while the system variables are not fully measurable. Among these systems we find the AD process, where the bacteria that are key parameters of the process are not accessible for measurement. To overcome this problem, a state observer can be included in the control design, allowing reconstruction of the state vector. However, the choice of the state observer is crucial since its dynamics affect the stability of the closed loop system.

Concerning the two-step models of the AD process, we propose to use the nonlinear observer designed in [7], due to its exponential convergence and systematic implementation. Moreover, to keep the results general and convenient for other nonlinear systems belonging to the same class of systems as the AD process, we represent the AD model by the following system

$$\begin{cases}\dot{x}=A(\rho^u)x+G\gamma(x)+Bu\\ y=Cx\end{cases} \tag{4}$$

where the state vector $x \in \mathbb{R}^n$, the input $u \in \mathbb{R}^q$ and the linear output measurements $y \in \mathbb{R}^p$. The parameter $\rho^u \in \mathbb{R}^s$ is an $\mathcal{L}_\infty$ bounded and known parameter and the affine matrix $A(\rho^u)$ is expressed under the form

$$A(\rho^u) = A_0 + \sum_{j=1}^{s} \rho_j A_j \tag{5}$$

with

$$\rho_{\min} \leq \rho^u \leq \rho_{\max} \tag{6}$$

which means that the parameter ρ^u belongs to a bounded convex set for which the set of 2^s vertices can be defined by:

$$\mathbb{V}_{\rho^u} = \left\{ \varrho \in \mathbb{R}^s : \varrho_j \in \{\rho_{j,\min}, \rho_{j,\max}\} \right\}. \tag{7}$$

The matrices $A_i \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are constant. The nonlinear function $\gamma : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is assumed to be globally Lipschitz and can always be written under the detailed form:

$$\gamma(x) = \sum_{i=1}^{m} \gamma_i(\overbrace{H_i x}^{\vartheta_i}) \tag{8}$$

where $H_i \in \mathbb{R}^{n_i \times n}$.

Remark 1. The parameter ρ^u depends only on the control input vector u. Nevertheless, it can depend also on the output y and any other exogenous variable. However, without loss of generality, we used only u, which is motivated by the anaerobic digestion model considered in this paper.

In order to reconstruct the state vector of the system (4), we use the following state observer:

$$\dot{\hat{x}} = A(\rho^u)\hat{x} + G\sum_{i=1}^{m} \gamma_i(\hat{\vartheta}_i) + Bu + L(\rho^u)\left(y - C\hat{x}\right) \tag{9}$$

with

$$\hat{\vartheta}_i = H_i\hat{x} + \mathcal{K}_i(\rho^u)(y - C\hat{x}), \tag{10}$$

and

$$L(\rho^u) = L_0 + \sum_{j=1}^{s} \rho_j^u L_j, \;\; \mathcal{K}_i(\rho^u) = \mathcal{K}_{i0} + \sum_{j=1}^{s} \rho_j^u \mathcal{K}_{ij}. \tag{11}$$

where $\hat{x}$ is the estimate of the system state x. The matrices $L_i \in \mathbb{R}^{n\times p}$ and $\mathcal{K}_{ij} \in \mathbb{R}^{n_i \times p}$ are the observer gains, to be computed so that the estimation error

$$e = x - \hat{x} \tag{12}$$

turns to be exponentially stable.

Since $\gamma(.)$ is globally Lipschitz, then according to Lemma 1.1 there exist $z_i \in Co(\vartheta_i, \hat{\vartheta}_i)$, functions $\phi_{ij} : \mathbb{R}^{n_i} \longrightarrow \mathbb{R}$, and constants a_{ij}, b_{ij}, so that

$$G(\gamma(x) - \gamma(\hat{x})) = \sum_{i,j=1}^{m,n_i} \phi_{ij}(z_i)\mathcal{H}_{ij}\left(\vartheta_i - \hat{\vartheta}_i\right) \tag{13}$$

and

$$a_{ij} \leq \phi_{ij}(z_i) \leq b_{ij}, \tag{14}$$

where

$$\phi_{ij}(z_i) = \frac{\partial \gamma_i}{\partial \vartheta_i^j}(z_i), \;\; \mathcal{H}_{ij} = Ge_m^T(i)e_{n_i}(j) \tag{15}$$

For shortness, we set $\phi_{ij} \triangleq \phi_{ij}(z_i)$. Without loss of generality, we assume that $a_{ij} = 0$ for all $i = 1, \ldots, m$ and $j = 1, \ldots, n_i$. For more details about this, we refer the reader to [1].

Since $\vartheta_i - \hat{\vartheta}_i = (H_i - \mathcal{K}_i(\rho^u)C)e$, then dynamics of the estimation error are given by

$$\dot{e} = \left(\mathbb{A}_L(\rho^u) + \sum_{i,j=1}^{m,n_i} \phi_{ij}\mathcal{H}_{ij}\left(H_i - \mathcal{K}_i(\rho^u)C\right)\right)e \tag{16}$$

with

$$\mathbb{A}_L(\rho^u) = A(\rho^u) - L(\rho^u)C \tag{17}$$

As already explained, dynamics of the estimation error affect the stability of the closed loop system. Thus, before discussing the stability analysis of the chosen observer and the computation of its parameters, we first present the observer based trajectory tracking control. Then, we discuss the stability conditions of the closed loop system composed of the system, the observer and the controller.

A feasible trajectory for the system (4) is a pair (x_d, u_d) that satisfies the differential equation and generates the desired trajectory:

$$\begin{cases} \dot{x}_d = A(\rho^{u_d})x_d + G\gamma(x_d) + Bu_d \\ y_d = Cx_d \end{cases} \tag{18}$$

where x_d and u_d represent the desired state and the desired input, respectively. Assuming that it is possible to find a feasible trajectory for the system, then it is possible to search for controllers $u = f(x, x_d, u_d)$ that track the desired trajectory. Besides, due to the absence of key measurements, we use the state estimate instead. That is the tracking control reads

$$u = -K(\rho^{u_d})(\hat{x} - x_d) + u_d \tag{19}$$

where

$$K(\rho^{u_d}) = K_0 + \sum_{j=1}^{s} \rho_j^{u_d} K_j \tag{20}$$

Let us define the tracking error by

$$\tilde{x} = x - x_d \tag{21}$$

Its dynamic can be easily obtained as

$$\begin{aligned} \dot{\tilde{x}} = & \left(A(\rho^u) - BK(\rho^{u_d}) + \sum_{i,j=1}^{m,n_i} \varphi_{ij}(t)\mathcal{H}_{ij}H_i\right)\tilde{x} + BK(\rho^{u_d})e \\ & + \underbrace{(A(\rho^u) - A(\rho^{u_d}))x_d}_{\omega(t)} \end{aligned} \tag{22}$$

where $\varphi_{ij} \triangleq \frac{\partial \gamma_i}{\partial \vartheta_i^j}(\nu_i)$, with $\nu_i \in Co(x, x_d)$ and

$$\underline{\varphi}_{ij} \leq \varphi_{ij} \leq \overline{\varphi}_{ij} \tag{23}$$

The aim consists of finding the controller and observer gain matrices, so that the tracking error $\tilde{x}$ satisfies the following $\mathcal{H}_\infty$ criterion

$$\|\tilde{x}\|_{\mathcal{L}_2^n} \leq \sqrt{\mu\|\omega\|^2_{\mathcal{L}_2^n} + \eta\left\|\begin{pmatrix} \tilde{x}_0 \\ e_0 \end{pmatrix}\right\|^2} \tag{24}$$

where $\mu > 0$ is the gain from ω to $\tilde{x}$, and $\eta > 0$. In the next section, we provide the stability conditions that satisfy this objective.

3. New LMI procedure to design the observer-based controller

In this section we present a kind of separation principle for nonlinear systems. Since the dynamics (16) do not depend on the reference tracking error $\tilde{x}$ and the functions ϕ_{ij} are bounded, then we can study the convergence of the estimation error e separately, and use it in dynamics of the tracking error as a bounded disturbance exponentially converging towards zero. The following theorem provides the synthesis conditions expressed in term of LMIs.

3.1. Design of the observer gains

We first state a theorem which provides the LMI conditions ensuring the exponential convergence of the estimation error towards zero.

Theorem 3.1. *For a given positive scalar β, if there exist symmetric positive definite matrices $\mathbb{Q}$, $\mathcal{S}_i$, $i = 1, \dots, n$, and matrices $\mathbb{X}_i$, $\mathcal{X}_{ij}$, $i = 1, \dots, n$ and $j = 1, \dots, s$, of appropriate dimensions, such that the following LMI conditions (25) are fulfilled for system (4):*

$$\begin{bmatrix} \mathbb{A}\Big(\mathbb{Q}, \mathbb{X}, \varrho\Big) + \beta\mathbb{Q} & \overbrace{\begin{bmatrix} \Pi_1 & \dots & \Pi_m \end{bmatrix}}^{\Pi} \\ (\star) & -\Lambda\mathbb{S} \end{bmatrix} \leq 0, \ \forall \varrho \in \mathbb{V}_\rho \tag{25}$$

then the observer (9)–(11) with gains

$$L_j = \mathbb{Q}^{-1}\mathbb{X}_j^T, \quad \mathcal{K}_{ij} = \mathcal{S}_i^{-1}\mathcal{X}_{ij}^T \tag{26}$$

guarantees the exponential convergence of the estimation error $e(t)$, following the inequality:

$$\|e(t)\| \leq \sqrt{\frac{\lambda_{\max}(\mathbb{Q})}{\lambda_{\min}(\mathbb{Q})}}\|e_0\|e^{-\frac{\beta}{2}t}, \tag{27}$$

where

$$\mathbb{A}\Big(\mathbb{Q}, \mathbb{X}, \varrho\Big) = A_0^T\mathbb{Q} + \mathbb{Q}A_0 - C^T\mathbb{X}_0 - \mathbb{X}_0^T C + \sum_{j=1}^{s} \varrho_j\Big(A_j^T\mathbb{Q} + \mathbb{Q}A_j - C^T\mathbb{X}_j - \mathbb{X}_j^T C\Big) \tag{28}$$

$$\Pi_i = \Big[\mathcal{M}_i^1\Big(\mathbb{Q}, \mathcal{S}_i\Big) \dots \mathcal{M}_i^{n_i}\Big(\mathbb{Q}, \mathcal{S}_i\Big)\Big] \tag{29}$$

$$\mathcal{M}_i^j\Big(\mathbb{Q}, \mathcal{S}_i\Big) = \mathbb{Q}\mathcal{H}_{ij} + H_i^T\mathcal{S}_i - C^\top\Big(\mathcal{X}_{i0} + \sum_{j=1}^{s}\varrho_j\mathcal{X}_{ij}\Big) \tag{30}$$

$$\Lambda = \text{block-diag}\Big(\Lambda_1, \dots, \Lambda_m\Big) \tag{31}$$

$$\Lambda_i = \text{block-diag}\Bigg(\frac{2}{\bar{\varphi}_{i1}}\mathbb{I}_{n_i}, \dots, \frac{2}{\bar{\varphi}_{in_i}}\mathbb{I}_{n_i}\Bigg) \tag{32}$$

$$\mathbb{S} = \text{block-diag}\Big(\mathbb{S}_1, \dots, \mathbb{S}_m\Big) \tag{33}$$

$$\mathbb{S}_i = \text{block-diag}\Big(\mathcal{S}_i, \dots, \mathcal{S}_i\Big). \tag{34}$$

Proof. Consider the quadratic Lyapunov function

$$V(e) = e^T\mathbb{Q}e, \quad \mathbb{Q} = \mathbb{Q}^T > 0. \tag{35}$$

Then, its derivative $\dot{V}(e)$ along the trajectories (16) is given by

$$\dot{V}(e) = e^T\Bigg[\Bigg(\mathbb{A}_L(\rho) + \sum_{i,j=1}^{m,n_i}\phi_{ij}\mathcal{H}_{ij}\mathbb{H}_{K_i}\Bigg)^T\mathbb{Q} + \mathbb{Q}\Bigg(\mathbb{A}_L(\rho) + \sum_{i,j=1}^{m,n_i}\phi_{ij}\mathcal{H}_{ij}\mathbb{H}_{K_i}\Bigg)\Bigg]e \tag{36}$$

with

$$\mathbb{A}_L = A(\rho) - L(\rho)C, \quad \mathbb{H}_{K_i} = H_i - K_i(\rho)C \tag{37}$$

The derivative of Lyapunov function (36) is negative if

$$\mathbb{A}_L^T(\rho)\mathbb{Q} + \mathbb{Q}\mathbb{A}_L(\rho) + \sum_{i,j=1}^{m,n_i}\phi_{ij}\Bigg(\underbrace{\mathbb{Q}\mathcal{H}_{ij}}_{\mathbb{X}_{ij}^T}\underbrace{\mathbb{H}_{K_i}}_{\mathbb{Y}_i} + \mathbb{Y}_i^T\mathbb{X}_{ij}\Bigg) < 0 \tag{38}$$

From Lemma 1.2, we know that for any symmetric positive definite matrices $\mathbb{S}_{ij}$, we have

$$\mathbb{X}_{ij}^T\mathbb{Y}_i + \mathbb{Y}_i^T\mathbb{X}_{ij} \leq \frac{1}{2}\Big[\mathbb{X}_{ij} + \mathbb{S}_{ij}\mathbb{Y}_i\Big]^T\mathbb{S}_{ij}^{-1}\underbrace{\Big[\mathbb{X}_{ij} + \mathbb{S}_{ij}\mathbb{Y}_i\Big]}_{\mathcal{M}_i^j\left(\mathbb{Q}, \mathbb{S}_{ij}\right)} \tag{39}$$

Since the matrix $\mathbb{Y}_i$ does not depend on the index j and depends on the same $K_i(\rho_k)$, then to get an LMI, we need to put

$$\mathbb{S}_{ij} = \mathcal{S}_i, \qquad \forall (i, j) \tag{40}$$

Consequently, from (23) and the fact that without loss of generality $a_{ij} = 0$, inequality (38) is satisfied if

$$\mathbb{A}_L^T(\rho)P + P\mathbb{A}_L(\rho) + \sum_{i,j=1}^{m,n_i}\Bigg(\mathcal{M}_i^j\Big(\mathbb{Q}, \mathcal{S}_i\Big)^T\Big(-\frac{2}{b_{ij}}\mathcal{S}_i\Big)^{-1}\mathcal{M}_i^j\Big(\mathbb{Q}, \mathcal{S}_i\Big)\Bigg) \leq 0. \tag{41}$$

Therefore, from Schur lemma and with the change of variables $\mathbb{X}_j = L_j^T\mathbb{Q}$ and $\mathcal{X}_{ij} = \mathcal{K}_{ij}^T\mathcal{S}_i$, inequality (41) is equivalent to

$$\begin{bmatrix} \mathbb{A}_L^T(\rho)P + P\mathbb{A}_L(\rho) & \begin{bmatrix} \Pi_1 & \dots & \Pi_m \end{bmatrix} \\ (\star) & -\Lambda\mathbb{S} \end{bmatrix} \leq 0. \tag{42}$$

Since (42) is affine in ρ^u, then from the convexity principle [5] and the notations (28)–(34) we deduce that the LMIs (25) guarantees the inequality

$$\dot{V}(e) + \beta V(e) \leq 0, \tag{43}$$

which leads to

$$V(e) \leq V(e_0)e^{-\beta t}. \tag{44}$$

From the fact that $V(e)$ satisfies the inequality

$$\lambda_{\min}(\mathbb{Q})\|e(t)\|^2 \leq V(e(t)) \leq \lambda_{\max}(\mathbb{Q})\|e_0\|^2 e^{-\beta t} \tag{45}$$

then we get the exponential convergence of the estimation error as follows:

$$\|e(t)\| \leq \sqrt{\frac{\lambda_{\max}(\mathbb{Q})}{\lambda_{\min}(\mathbb{Q})}}\|e_0\|e^{-\frac{\beta}{2}t}, \tag{46}$$

where $\frac{\beta}{2}$ is the convergence rate of the error $e(t)$. This ends the proof of Theorem 3.1. □

3.2. Design of the controller parameters

Hereafter, we present a second theorem, which provides new LMI conditions constituting the second step of the proposed observer-based controller synthesis methodology.

Theorem 3.2. *If there exist symmetric positive definite matrices $\mathbb{P}$, $\mathcal{Z}_{ij}$, $i, j = 1, \dots, n$, and matrices $\mathbb{Y}_i$ of appropriate dimensions, such that the convex optimization problem (47) is solvable:*

$$\min(\mu) \quad \text{subject to} \quad (48) \tag{47}$$

$$\begin{bmatrix} \Theta & \overbrace{\begin{bmatrix} \Sigma_1 & \dots & \Sigma_m \end{bmatrix}}^{\Sigma} \\ (\star) & -\Lambda\mathbb{Z} \end{bmatrix} \leq 0, \ \forall \varrho \in \mathbb{V}_\rho \tag{48}$$

then the controller (19) and (20) with gains

$$K_j = \mathbb{Y}_j^\top \mathbb{P}^{-1}, \ \ j = 1, \dots, s, \tag{49}$$

guarantees the following $\mathcal{H}_\infty$ performance criterion on the tracking error $\tilde{x}$:

$$\|\tilde{x}\|_{\mathcal{L}_2^n} \leq \sqrt{\mu \left\| \begin{pmatrix} \omega \\ K(\rho^{u_d})e \end{pmatrix} \right\|^2_{\mathcal{L}_2^n} + \lambda_{\max}(\mathbb{P})\|\tilde{x}_0\|^2}, \tag{50}$$

where

$$\Theta = \begin{bmatrix} \Theta_{11} & \begin{bmatrix} \mathbb{P} \\ 0 \end{bmatrix} \\ (\star) & -\mathbb{I}_n \end{bmatrix}, \quad \Theta_{11} = \begin{bmatrix} \mathbb{A}\left(\mathbb{P}, \mathbb{Y}, \varrho\right) & [\mathbb{I}_n \quad B] \\ (\star) & -\mu\mathbb{I}_{n+q} \end{bmatrix}$$

$$\mathbb{A}\left(\mathbb{P}, \mathbb{Y}, \varrho\right) = \mathbb{P}A_0^T + A_0\mathbb{P} - \mathbb{Y}_0 B^\top - B\mathbb{Y}_0^\top + \sum_{j=1}^{s} \varrho_j \left(\mathbb{P}A_j^T + A_0\mathbb{P} - \mathbb{Y}_j B^\top - B\mathbb{Y}_j^\top\right) \tag{51}$$

$$\Sigma_i = \left[\mathcal{N}_i^1\left(\mathbb{P}, \mathcal{Z}_{i1}\right) \dots \mathcal{N}_i^{n_i}\left(\mathbb{P}, \mathcal{Z}_{in_i}\right)\right] \tag{52}$$

$$\mathcal{N}_i^j\left(\mathbb{P}, \mathcal{Z}_{ij}\right) = \begin{bmatrix} \mathbb{P}H_i^T \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \mathcal{H}_{ij} \\ 0 \\ 0 \end{bmatrix} \mathcal{Z}_{ij} \tag{53}$$

$$\Lambda = \text{block-diag}\left(\Lambda_1, \dots, \Lambda_m\right) \tag{54}$$

$$\Lambda_i = \text{block-diag}\left(\frac{2}{\bar{\varphi}_{i1}}\mathbb{I}_{n_i}, \dots, \frac{2}{\bar{\varphi}_{in_i}}\mathbb{I}_{n_i}\right) \tag{55}$$

$$\mathbb{Z} = \text{block-diag}\left(\mathbb{Z}_1, \dots, \mathbb{Z}_m\right) \tag{56}$$

$$\mathbb{Z}_i = \text{block-diag}\left(\mathcal{Z}_{i1}, \dots, \mathcal{Z}_{in_i}\right). \tag{57}$$

Proof. Consider the Lyapunov function

$$V(\tilde{x}) = \tilde{x}^T \mathbb{P}^{-1} \tilde{x}. \tag{58}$$

By calculating the derivative of $V(\tilde{x})$ along the trajectories of (22), we get

$$\dot{V} + \tilde{x}^T\tilde{x} - \mu\bar{\omega}^T\bar{\omega} = \begin{pmatrix} \tilde{x} \\ \bar{\omega} \end{pmatrix} \begin{bmatrix} \mathbb{P}^{-1}\mathbb{D} + \mathbb{D}^T\mathbb{P}^{-1} + \mathbb{I}_n & \mathbb{P}^{-1}\bar{B} \\ (\star) & -\mu\mathbb{I}_{n+q} \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \bar{\omega} \end{pmatrix}^T \tag{59}$$

with

$$\mathbb{D} = A(\rho^{u_d}) - BK(\rho^{u_d}) + \sum_{i,j=1}^{m,n_i} \varphi_{ij}(t)\mathcal{H}_{ij}H_i, \tag{60}$$

$$\bar{B} = [\mathbb{I}_n \quad B] \ \text{ and } \ \bar{\omega} = \begin{pmatrix} \omega \\ K(\rho^{u_d})e \end{pmatrix}. \tag{61}$$

Inequality (59) holds if the following matrix is positive definite

$$\begin{bmatrix} \mathbb{P}^{-1}\mathbb{D} + \mathbb{D}^T\mathbb{P}^{-1} + \mathbb{I}_n & \mathbb{P}^{-1}\bar{B} \\ (\star) & -\mu\mathbb{I}_{n+q} \end{bmatrix} < 0, \tag{62}$$

which is equivalent to

$$\begin{bmatrix} \mathbb{D}\mathbb{P} + \mathbb{P}\mathbb{D}^T + \mathbb{P}^2 & \bar{B} \\ (\star) & -\mu\mathbb{I}_{n+q} \end{bmatrix} < 0 \tag{63}$$

by pre- and post-multiplying (62) by

$$\begin{bmatrix} \mathbb{P} & 0 \\ 0 & \mathbb{I}_n \end{bmatrix}.$$

Using the Schur lemma, it follows that inequality (63) is equivalent to

$$\begin{bmatrix} \mathbb{D}\mathbb{P} + \mathbb{P}\mathbb{D}^T & \bar{B} & \mathbb{P} \\ (\star) & -\mu\mathbb{I}_{n+q} & 0 \\ (\star) & 0 & -\mathbb{I}_n \end{bmatrix} < 0, \tag{64}$$

which can be rewritten under the form:

$$\begin{bmatrix} (A(\rho^{u_d}) - BK(\rho^{u_d}))\mathbb{P} + \mathbb{P}(A(\rho^{u_d}) - BK(\rho^{u_d}))^T & \bar{B} & \mathbb{P} \\ (\star) & -\mu\mathbb{I}_{n+q} & 0 \\ (\star) & 0 & -\mathbb{I}_n \end{bmatrix} + \sum_{i,j=1}^{m,n_i} \varphi_{ij}(t) \left(\overbrace{\begin{bmatrix} \mathbb{P}H_i^T \\ 0 \\ 0 \end{bmatrix}}^{\mathbb{X}_i^T} \overbrace{\begin{bmatrix} \mathcal{H}_{ij}^T & 0 & 0 \end{bmatrix}}^{\mathbb{Y}_{ij}} + \mathbb{Y}_{ij}^T\mathbb{X}_i \right) < 0. \tag{65}$$

From Lemma 1.2, for any symmetric positive definite matrices $\mathbb{Z}_{ij}$, the inequality (65) holds if the following one is fulfilled:

$$\begin{bmatrix} (A(\rho^{u_d}) - BK(\rho^{u_d}))\mathbb{P} + \mathbb{P}(A(\rho^{u_d}) - BK(\rho^{u_d}))^T & \bar{B} & \mathbb{P} \\ (\star) & -\mu\mathbb{I}_{n+q} & 0 \\ (\star) & 0 & -\mathbb{I}_n \end{bmatrix} + \sum_{i,j=1}^{m,n_i} \frac{\varphi_{ij}(t)}{2} \left[\mathbb{X}_i + \mathcal{Z}_{ij}\mathbb{Y}_{ij}\right]^T \mathcal{Z}_{ij}^{-1} \underbrace{\left[\mathbb{X}_i + \mathcal{Z}_{ij}\mathbb{Y}_{ij}\right]}_{\mathcal{N}_i^j\left(\mathbb{P}, \mathcal{Z}_{ij}\right)} < 0. \tag{66}$$

Therefore, using the notation of Theorem 3.2, applying the Schur lemma, and from the convexity principle, the inequality (66) is equivalent to (48). This means that

$$\dot{V}(\tilde{x}) + \tilde{x}^T\tilde{x} - \mu\bar{\omega}^T\bar{\omega} \leq 0, \tag{67}$$

which leads to (50) by integrating (67) from 0 to $+\infty$. Indeed, by integrating (67), we get

$$V(+\infty) - V(0) + \int_0^{+\infty} \tilde{x}^T(t)\tilde{x}(t)\text{d}t - \mu\int_0^{+\infty} \bar{\omega}^T(t)\bar{\omega}(t)\text{d}t \leq 0$$

Since $V(+\infty) > 0$ and $V(0) \leq \lambda_{\max}(\mathbb{P})\|\tilde{x}_0\|^2$, then we have

$$\int_0^{+\infty} \tilde{x}^T(t)\tilde{x}(t)\text{d}t \leq \mu\int_0^{+\infty} \bar{\omega}^T(t)\bar{\omega}(t)\text{d}t + \lambda_{\max}(\mathbb{P})\|\tilde{x}_0\|^2$$

which is equivalent to (50).This ends the proof. □

3.3. Fulfilment of the $\mathcal{H}_\infty$ criterion (24)

Now we complete the convergence analysis part of the proposed $\mathcal{H}_\infty$ observer-based tracking design methodology. The last and final step of the convergence proof can be summarized in the next proposition.

Proposition 3.1. *If there exist symmetric positive definite matrices $\mathbb{P}$, $\mathbb{Q}$, $\mathcal{Z}_{ij}$, $\mathcal{S}_i$, $i, j = 1, \dots, n$, matrices $\mathbb{Y}_i$, $\mathbb{X}_i$, $\mathcal{X}_{ij}, i = 1, \dots, n; j = 1, \dots, s$ of appropriate dimensions, and a positive scalar β, such that the following two items hold:*

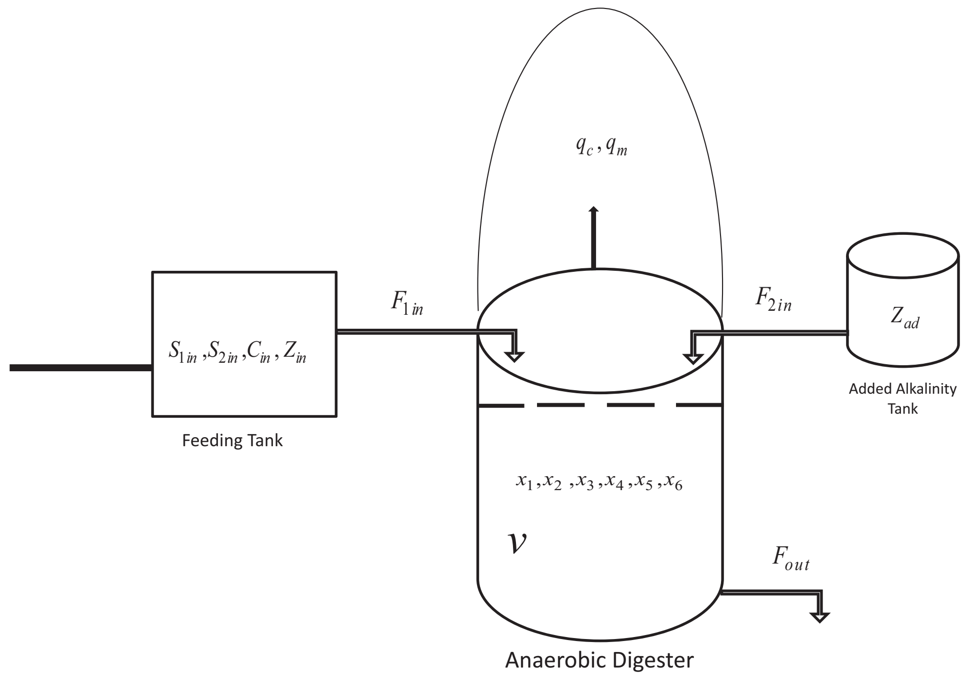

Fig. 1. Controlled anaerobic digestion process.

(i) *The LMI conditions (25) are fulfilled;*
(ii) *the convex optimization problem (47) is solvable,*

then, the observer-based controller (19) and (20) with gains given by (26) and (49), guarantees the $\mathcal{H}_\infty$ criterion (24) on $\tilde{x}$ with μ returned by (47) and η given by

$$\eta \triangleq \max\left(\lambda_{\max}(\mathbb{P}),\ \frac{1}{\beta}\frac{\lambda_{\max}(\mathbb{Q})\lambda_{\max}(K^\top(\rho^{u_d})K(\rho^{u_d}))}{\lambda_{\min}(\mathbb{Q})}\right). \tag{68}$$

Proof. Since LMIs (25) hold, then the estimation error satisfies (27). Then, we have

$$\begin{aligned}\left\|\begin{pmatrix}\omega\\ K(\rho^{u_d})e\end{pmatrix}\right\|^2_{\mathcal{L}_2^n} &\le \|\omega\|^2_{\mathcal{L}_2^n} + \lambda_{\max}\left(K^\top(\rho^{u_d})K(\rho^{u_d})\right)\|e\|^2_{\mathcal{L}_2^n}\\ &\le \|\omega\|^2_{\mathcal{L}_2^n} + \lambda_{\max}\left(K^\top(\rho^{u_d})K(\rho^{u_d})\right)\frac{\lambda_{\max}(\mathbb{Q})}{\lambda_{\min}(\mathbb{Q})}\|e_0\|^2\int_0^{+\infty} e^{-\beta\tau}\,\mathrm{d}\tau\\ &= \|\omega\|^2_{\mathcal{L}_2^n} + \frac{\lambda_{\max}\left(K^\top(\rho^{u_d})K(\rho^{u_d})\right)}{\beta}\frac{\lambda_{\max}(\mathbb{Q})}{\lambda_{\min}(\mathbb{Q})}\|e_0\|^2.\end{aligned} \tag{69}$$

On the other hand, the solvability of the convex optimization problem (47) means that the performance criterion(50) holds. It follows that

$$\|\tilde{x}\|_{\mathcal{L}_2^n} \le \sqrt{\mu\|\omega\|^2_{\mathcal{L}_2^n} + \max\left(\lambda_{\max}(\mathbb{P}),\ \frac{\lambda_{\max}(\mathbb{Q})\lambda_{\max}(K^\top(\rho^{u_d})K(\rho^{u_d}))}{\beta\lambda_{\min}(\mathbb{Q})}\right)\left\|\begin{pmatrix}\tilde{x}_0\\ e_0\end{pmatrix}\right\|^2}. \tag{70}$$

This ends the proof of Proposition 3.1. □

Remark 2. The proposed LMI-based design technique is easy to implement numerically. Indeed, once the parameters of a given model are known, it suffices to solve the proposed LMIs to get the observer-based controller gains ensuring the $\mathcal{H}_\infty$ criterion. To solve such LMIs, we use some available numerical software toolboxes like Matlab LMI Toolbox by using YALMIP. As for the computational aspect and the implementation complexity, there are no obstacles for real-time applications because the observer-based controller parameters are computed offline by solving the LMIs.

4. Application to an anaerobic digestion process

4.1. Mathematical model of the AD process

AD modelling has been widely investigated in the literature. Often, the resulted model is driven by the application objectives, waste nature and its composition, the available data and its reliability. In this paper, being motivated by the control and observer design for the AD processes, we will use the same model considered in [7] which is based on the AM2 model [3] with the inclusion of one additional input representing the addition of stimulating alkalinity as shown in Fig. 1. The resulted model structure reads

$$\begin{cases}\dot{x}_1 = -k_1\mu_1(x_1)x_2 + u_1S_{1in} - ux_1\\ \dot{x}_2 = (\mu_1(x_1) - \alpha u)x_2\\ \dot{x}_3 = k_2\mu_1(x_1)x_2 - k_3\mu_2(x_3)x_4 + u_1S_{2in} - ux_3\\ \dot{x}_4 = (\mu_2(x_3) - \alpha u)x_4\\ \dot{x}_5 = k_4\mu_1(x_1)x_2 + k_5\mu_2(x_3)x_4 + u_1C_{in} - ux_5 - q_c(x)\\ \dot{x}_6 = u_1Z_{in} + u_2Z_{ad} - ux_6\end{cases} \tag{71}$$

$$\begin{cases}y_1 = q_c(x)\\ y_2 = [x_1, x_3, x_6]^T\end{cases} \tag{72}$$

with

$$\mu_1(x_1) = \overline{\mu}_1\frac{x_1}{x_1 + k_{s_1}}, \quad \mu_2(x_3) = \overline{\mu}_2\frac{x_3}{x_3 + k_{s2} + \frac{x_3^2}{k_{i2}}} \tag{73}$$

$$co_2 = x_5 + x_3 - x_6, \quad \phi = co_2 + K_HP_T + \frac{k_6}{k_{La}\mu_2(x_3)x_4} \tag{74}$$

$$q_c(x) = k_{La}[co_2 - K_HP_c(x)], \quad q_m(x) = k_6\mu_2(x_3)x_4 \tag{75}$$

$$P_c(x) = \frac{\phi - \sqrt{\phi^2 - 4K_HP_Tco_2}}{2K_H} \tag{76}$$

Table 1
Model parameters [3].

Acronyms	Definition	Units	Value
α	Proportion of dilution rate for bacteria	mmol/l	0.5
k_1	Yield for substrate (x_1) degradation	g/(g of x_2)	42.1
k_2	Yield for VFA (x_3) production	mmol/(g of x_2)	116.5
k_3	Yield for VFA consumption	mmol/(g of x_4)	268
k_4	Yield for co_2 production	mmol/g	50.6
k_5	Yield for co_2 production	mmol/g	343.6
k_6	Yield for ch_4 production	mmol/g	453
$\overline{\mu}_1$	Maximum acidogenic bacteria (x_2) growth rate	1/day	1.25
$\overline{\mu}_2$	Maximum methanogenic bacteria (x_4) growth rate	1/day	0.74
k_{s_1}	Half saturation constant associated with x_1	g/l	7.1
k_{s2}	Half saturation constant associated with x_3	mmol/l	9.28
k_{i2}	Inhibition constant associated with x_3	mmol/l	256
k_b	Acidity constant of bicarbonate	mol/l	6.5 10^{-7}
K_H	Henry's constant	mmol/(l.atm)	27
P_T	Total preasure	atm	1.013
k_{La}	Liquid/gas transfer constant	1/day	19.8

where x_1 (g/l) is the organic substrate concentration to be consumed by the acidogenic bacteria x_2 (g/l) for growth and production of volatile fatty acids x_3 (mmol/l) (assumed to behave like pure acetate), x_4 (g/l) is the methanogenic bacteria concentration, x_5 (mmol/l) represents the inorganic carbon concentration and x_6 (mmol/l) the alkalinity concentration. The related fed concentrations to the digester S_{1in}, S_{2in}, C_{in} and Z_{in} are supposed to be known and constant. The control inputs are $u_1 = \frac{F_{1in}}{v}$ ($1/day$) and $u_2 = \frac{F_{2in}}{v}$ ($1/day$), where F_{1in} is the waste feeding rate and F_{2in} is the input flow rate of the added alkalinity (Z_{ad}) to the digester. Since the later volume (v) is constant, then the output flow rate $u = u_1 + u_2$ (see Fig. 1). The produced biogas is assumed to be composed of methane $q_m(x)$ and co_2 ($q_c(x)$) gaseous. The later partial pressure is computed by $P_c(x)$. The rest of the used parameters in the model are defined in Table 1.

4.2. Correspondance between (4) and (71)–(72)

The AD model (71) and (72) can be easily written under the form (4) using the following parameters

$$s = 1, \quad n = 6, \quad m = 2, \quad n_1 = 2, \quad n_2 = 2, \quad n_3 = 4,$$

$$\rho = u, \quad A_0 = 0_{n\times n}, \quad A_1 = -\text{block-diag}(1, \alpha, 1, \alpha, 1, 1),$$

$$G = \begin{bmatrix} -k_1 & 1 & k_2 & 0 & k_4 & 0 \\ 0 & 0 & -k_3 & 1 & k_5 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}^T,$$

$$\gamma(x) = \begin{bmatrix} \mu_1(x_1)x_2, & \mu_2(x_3)x_4, & q_c(x) \end{bmatrix}^T,$$

$$B = \begin{bmatrix} S_{1in} & 0 & S_{2in} & 0 & C_{in} & Z_{in} \\ 0 & 0 & 0 & 0 & 0 & Z_{ad} \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$H_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

4.3. Simulation results

In this section, we present a numerical example where we apply the proposed observer based tracking control to the AD process. The purpose is to track an admissible reference reflecting an enhanced quality of biogas. Moreover, to be as close as possible to real life experiments, we apply an additive zero-centered Gaussian noise to the performed measurements.

To run the simulation, we use the parameter values given in Table 1, and we take $S_{1in} = 16$ g/l, $S_{2in} = 170$ mmol/l, $C_{in} = 76.15$ mmol/l, $Z_{in} = 200$ mmol/l, $Z_{ad} = 700$ mmol/l. Also, we initialize the system and the observer at $x(0) = [1.8, 0.4, 12, 0.7, 53.48, 55]^T$ and $\hat{x}_0 = [1.8, 0.6, 12, 0.3, 45, 55]^T$, respectively. Regarding the desired reference, it corresponds to the steady state $x_d = [1.95, 0.60, 5.4, 1.38, 242.8, 240.34]^T$ and the admissible reference input $u_d = [0.4966, 0.0436]^T$ (we note that the pair (x_d, u_d) satisfies the equations 4). Besides, to solve the LMI conditions given by Theorem 3.2, we choose $\rho_{min} = 0.1$ (1/day) and $\rho_{max} = 0.8$ (1/day).

After solving the LMI conditions (25) and the optimization problem (47), which have been found feasible for $\beta = 0.06$, and by using the LMI Toolbox of Matlab and the solver SeDuMi, we have obtained the results depicted in Figs. 2–10. In the first 8 figures, the blue curve refers to the system state and the produced biogas quality without applying control. The red curve represents the desired reference, the green curve refers to the trajectory when applying control, and the black curve represents estimate of the controlled state. As it can be seen from these figures, despite the large initial estimation error and the corrupted measurements, the observer converges to the simulated system.

Moreover, we can see clearly from the results that despite the attached noise to measurements, the closed loop system tracks the

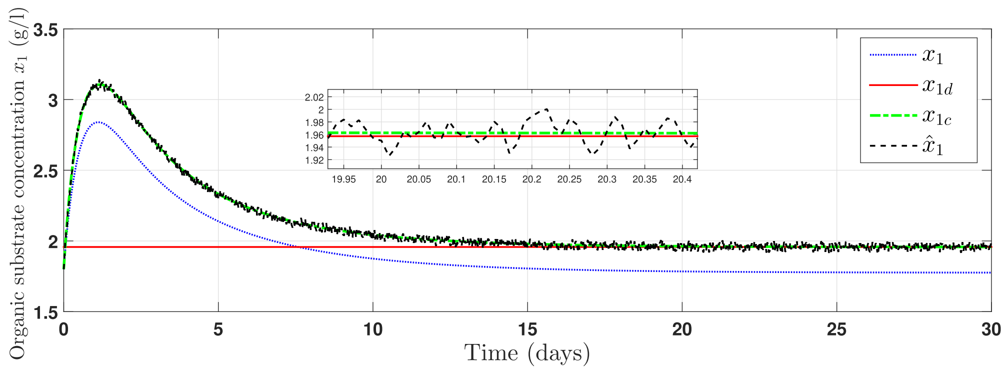

Fig. 2. Organic substrate concentration x_1 (g/l).

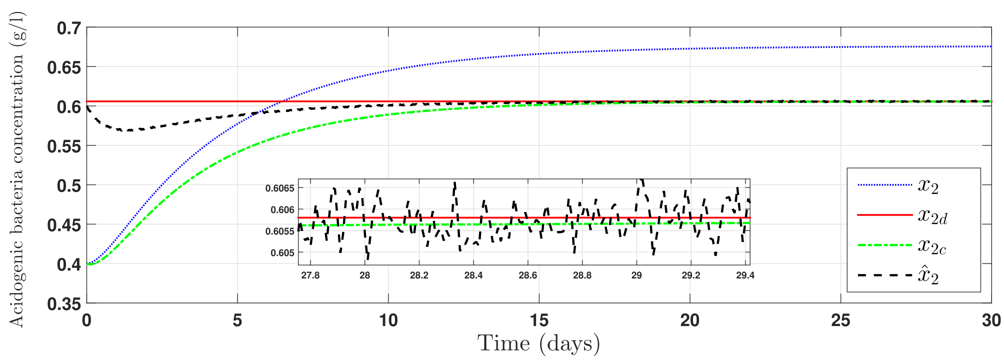

Fig. 3. Acidogenic bacteria concentration x_2 (g/l).

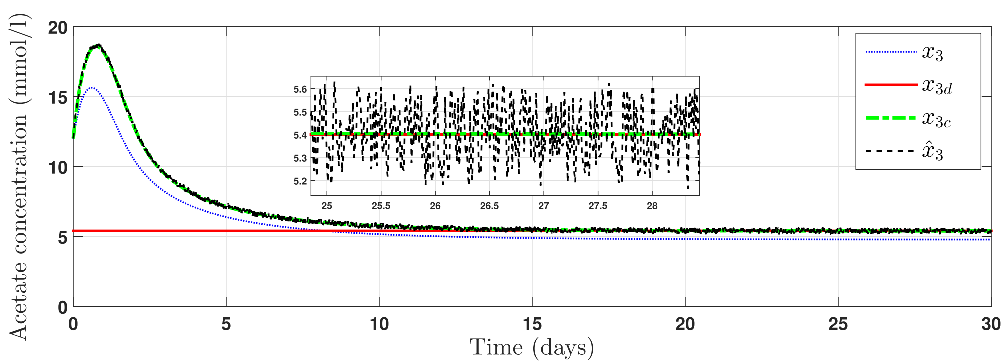

Fig. 4. Acetate concentration x_3 (mmol/l).

desired trajectory. Indeed, we can see from Fig. 8 that the biogas quality is enhanced by almost 10% when applying the designed control. This is a promising result from energy point of view of the process.

Last but not least, we want to emphasise that the controller behaviour, see Figs. 9 and 10, remains in an acceptable range of values. Indeed, in practice, for the well functioning of the digester and from the physical meaning of the state variables, variation of the control inputs is limited to avoid bacteria washout. This is, actually, what is obtained with the proposed observed based control strategy.

Remark 3. In a general situation, the upper and lower bounds of the parameter ρ^u are given by calculating the minimum and maximum values of the parameter. It is well know that given a bounded function, it is always possible to calculate the minimum and maximum values of such a function. These values are fixed (and constant) and depend on the parameter. On the other hand, in the case of the considered anaerobic digestion model, the parameter ρ^u depends on the input vector. Then, the values $\rho_{\min}$ and $\rho_{\max}$ are computed numerically by using the minimum and maximum values of the state vectors generated by the model. They can also depend on the saturated value of the control input if the saturation issue is considered. Consequently, the solutions of the LMIs depend on the saturated value. The LMIs may be feasible for a value and unfeasible for larger values (from the convexity principle). However, the technical analysis of this issue is one of the future work we aim to consider in a thorough way, from analytical and numerical points of view. We also aim to consider model uncertainties and noises with extended comparisons with other techniques from the literature.

Remark 4. Note that including a comparison of the results with another technique from the literature would be interesting and improve the paper. However, detailed comparisons are part of our future work. We aim to consider model uncertainties and process and measurement noises. On the other hand, some advantages of the proposed method compared to the MPC are obvious. MPC can work on thousands of different processes, especially for biogas. For instance, in [15], the authors proposed a model predictive

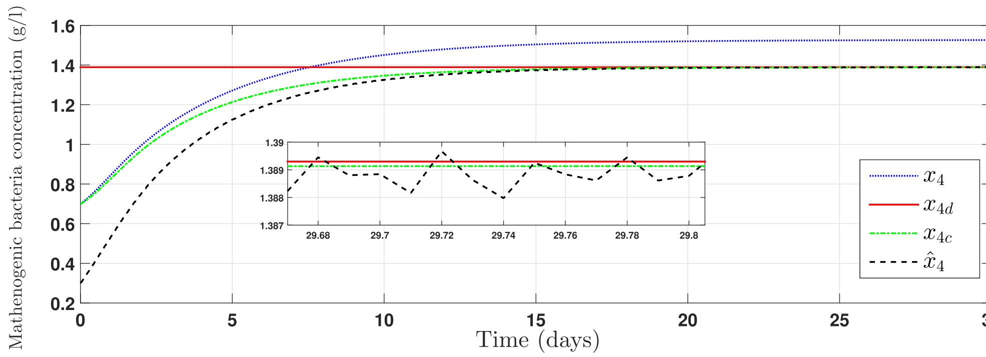

Fig. 5. Methanogenic bacteria concentration x_4 (g/l).

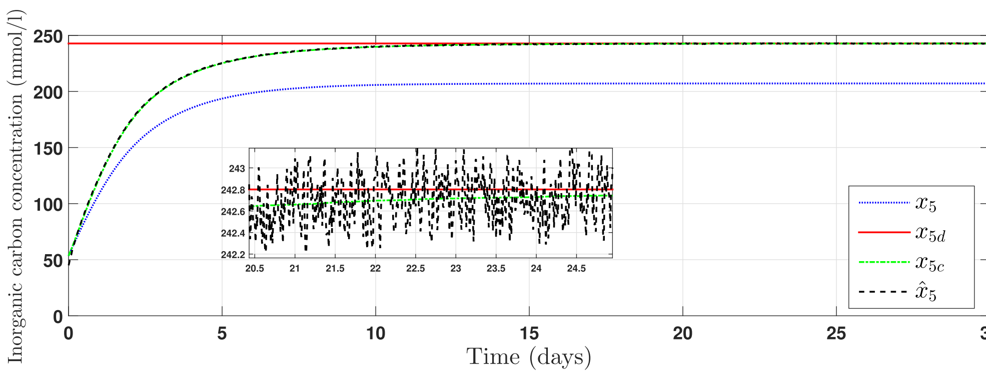

Fig. 6. Inorganic carbon concentration x_5 (mmol/l).

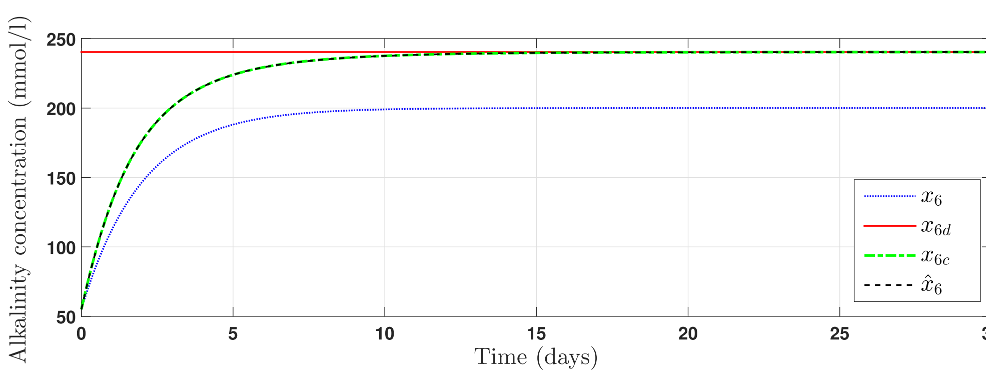

Fig. 7. Alkalinity concentration x_6 (mmol/l).

control for demand-driven biogas production, which works successfully. Their method may perform quite as well as the proposed LMI-based method. Nevertheless, they incur higher computational effort, because the MPC requires online computations, and there are some other drawbacks when compared to the proposed LMI-based method:

- Although MPC works on several processes, it has some drawbacks related to the stability when we face general nonlinear systems. On the other hand, the proposed LMI-based techniques overcome some MPC drawbacks, and guarantee global convergence for nonlinear systems.
- The computational complexity of MPC for real-time applications is very high compared to the proposed methodology where the LMIs are solved offline.
- Finding the MPC parameters ensuring local convergence is not always easy. For each model, we need to proceed with different MPC parameters that may be difficult to find for some complex models. On the other hand, with our LMI design method, it is sufficient to write the model under the considered structure and compute the bounds of the partial derivatives of the nonlinearities to solve the LMIs.

Remark 5. It should be noticed that the proposed method does not solve all the AD process issues. Motivated by the selected AD model, the aim of the paper is to consider the problem of observer-based trajectory tracking for a class of nonlinear systems using an LMI framework. Hence, new LMI conditions are proposed, and the result is applied to an AD process. Indeed, the paper does not consider robustness and disturbance rejections and startups/shutdowns, which are important issues in real-life applications. The theoretical analysis of these issues is one of the future work we aim to consider in a deepen way, from analytical and numerical points of view, with extended comparisons with other techniques from the literature.

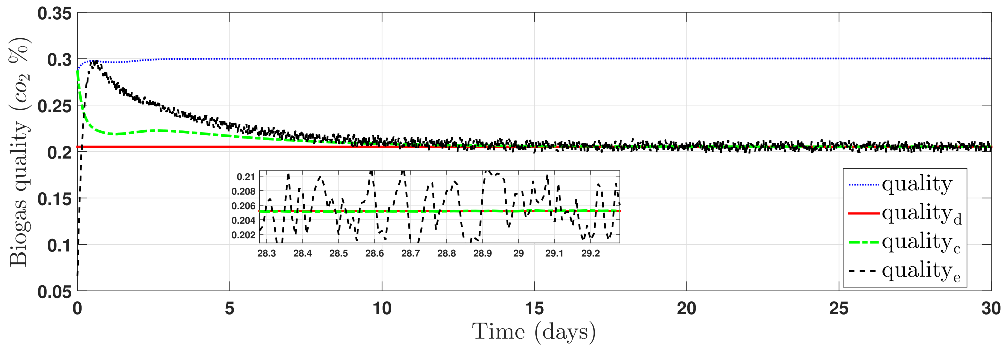

Fig. 8. Biogas quality (co_2%).

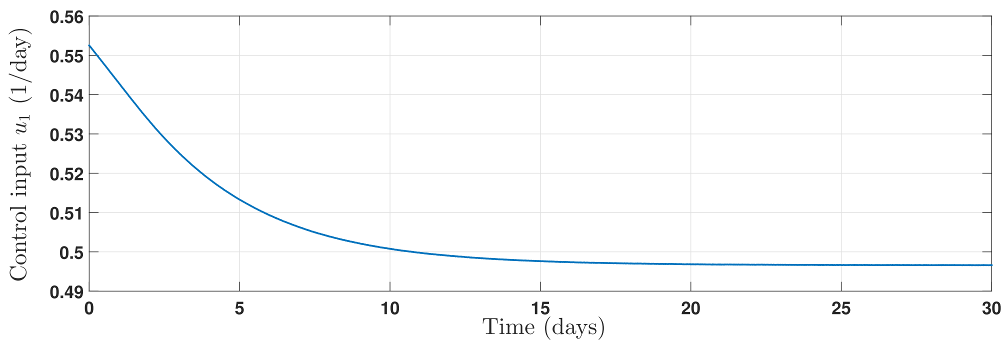

Fig. 9. Control input u_1 (1/day).

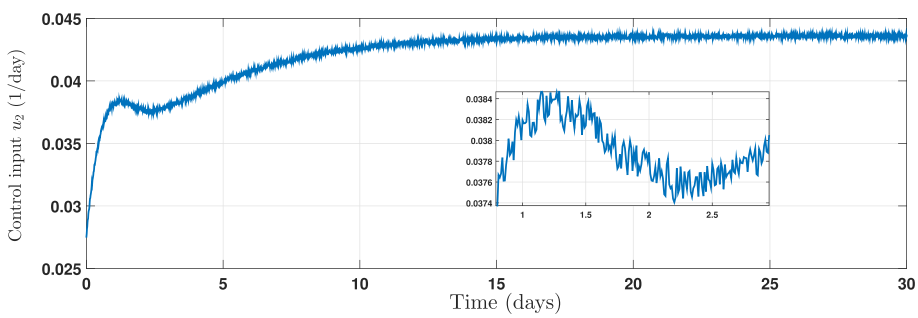

Fig. 10. Control input u_2 (1/day).

5. Conclusion

An observer-based reference tracking control for the AD process has been proposed in this paper. The control scheme is composed of an exponential nonlinear state observer and a feedback control. Actually, the state observer has been included in the control design to remedy the lack of some key measurements. Regarding the control law, feedback control has been used because it takes into account all dynamics of the process model.

In order to deal with the stability of the closed loop system, we have presented a kind of separation results. Indeed, relying on the triangular form of the system composed of dynamics of the estimation and tracking errors, we could establish the stability of each separately. Using an adequate reformulation of the Young's inequality and the Lipschitz property, we have provided the stability conditions in the form of enhanced end easily tractable LMIs. Firstly, to ensure the exponential stability of the estimation error. Then, to guarantee the H_∞ asymptotic stability of the tracking error.

We want to highlight that in order to extend the use of our technique and make it applicable for other systems belonging to the same class of systems as the AD process, we have presented the results in a general way. Furthermore, we have provided numerical simulations to illustrate the effectiveness of the proposed approach. In view of the simulation results, we target in the near future to extend the design methodology for saturation constraints on the control inputs. Moreover, as prospects, we will consider the disturbance rejections, startups/shutdowns, and parametric uncertainties, which are important issues in real-life applications. Extended comparisons to other available techniques in the literature would also increase the significance of the proposed design methodology.

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